IE 598: Games, Markets, and Mathematical Programming

Jugal Garg
- **Course webpage**
  - Grading Policy
  - Office Hours

- **Topics**
  - Games and Equilibrium Concepts
  - Routing Games and Price of Anarchy
  - Mechanism Design
  - Markets
Game Theory
Game Theory
Game Theory

Analysis/Design of a system where rational agents interact to achieve selfish goals
Multiple self-interested agents interacting in the same environment

Deciding what to do

Q: What to expect? How good is it? Can it be controlled?
Prisoner’s Dilemma

Two thieves caught for burglary
Two options: \{confess, remain silent\}
### Prisoners' Dilemma

<table>
<thead>
<tr>
<th>Prisoner A</th>
<th></th>
<th>Prisoner B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Confess</strong></td>
<td>5 years, 5 years</td>
<td>0 year, 20 years</td>
</tr>
<tr>
<td><strong>Remain silent</strong></td>
<td>20 years, 0 year</td>
<td>1 year, 1 year</td>
</tr>
</tbody>
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Prisoner’s Dilemma

Two thieves caught for burglary.
Two options: \{confess, remain silent\}

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<td>-5</td>
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<td>0</td>
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Prisoner’s Dilemma

Two thieves caught for burglary.
Two options: {confess, remain silent}

Only stable state
Tragedy of commons

Limited but open resource shared by many

Bad outcome!

Stable: Over use => Disaster
Rock-Paper-Scissors
Rock-Paper-Scissors

- Scissors beats paper
- Rock beats scissors
- Paper beats rock
No pure stable state!

Both playing \((1/3,1/3,1/3)\) is the only stable state.

Why?
Chicken
Routing Games and Price of Anarchy
Braess’ Paradox

100 commuters

Commute time: 1.5 hours
Braess’ Paradox

100 commuters

Commute time: 1.5 hours
Braess’ Paradox

100 commuters

Commute time: 2 hours
Braess’ Paradox

100 commuters

Price of Anarchy: \[\frac{\text{worst stable outcome}}{\text{OPT}} = \frac{2}{1.5} = \frac{4}{3}\]

Can’t be worse!
True Stories

• In **Stuttgart, Germany**, after investments into the road network in 1969, the traffic situation did not improve until a section of newly built road was closed for traffic again.

• In 1990 the temporary closing of 42nd Street in **New York City** for Earth Day reduced the amount of congestion in the area.

• In 2009, New York experimented with closures of **Broadway** at **Times Square** and **Herald Square**, which resulted in improved traffic flow and permanent pedestrian plazas.

...  

Source: Wikipedia.org
Single Item Auction
Vickery Auction

- Sealed bid auction
- Highest bidder wins and pays the second highest bid

Properties: Truthful, Incentive Compatible, Welfare Maximization
Adwords Auction

![Google search results](image)

- **Lauara Yoga Studio – Yoga & Massage Therapy**
  - [www.lauaryoga.com](http://www.lauaryoga.com)
  - "I knew nothing about yoga until I went here. Now I feel great and my family is into it too."
  - Lauara Yoga Studios – 34 Watts St. Denver, CO

- **Youth Yoga Classes in Denver, Colorado**
  - [www.yogakids.com](http://www.yogakids.com)
  - Yoga is for all ages! We offer youth yoga classes for kids after school, and on evenings and weekends. Modern facilities, reasonable rates.
  - Yoga Kids Inc. – 610 McKenzie Boul. Denver, CO

- **Hot Yoga Classes**
  - [www.yogabears.com/hotyoga](http://www.yogabears.com/hotyoga)
  - Dynamic, fun and cost effective! Special: 10 classes for $100

- **Yoga for beginners**
  - [www.vinashiyoga.com](http://www.vinashiyoga.com)
  - Burn calories and find peace. Small classes. First week free!
  - (354) 555-0111 - Directions

- **Lilac Yoga Studio**
  - [www.lilacyogadenver.com](http://www.lilacyogadenver.com)
  - Try our popular yoga sessions
  - Limited time $100 for 10!
Markets
At equilibrium:
Demand = Supply

A Single Good

Equilibrium

$P^*$

$Q^*$

Quantity

Price

Demand

Supply

Equilibrium
More than one good

Demand of a good now depend on price of other goods
Markets

Buyers have preferences over goods
want best affordable bundle

demand $\geq$ supply?
demand $< supply?$

Q: Does there exist an equilibrium?
If yes, how to compute it?
Do buyers have incentive to lie about their preferences?
Each buyer wants to buy exactly one house which maximizes valuation – payment and affordable
$v_{ij}$: valuation of buyer $i$ for house $j$

$$x_{ij} = \begin{cases} 1 & \text{if } j^{th} \text{ house is allocated to buyer } i \\ 0 & \text{otherwise} \end{cases}$$

$p_j$: price of house $j$

Maximize $\sum_{i,j} v_{ij} x_{ij}$

$$\sum_j x_{ij} \leq 1, \quad \forall i$$

$$\sum_i x_{ij} \leq 1, \quad \forall j$$

$x_{ij} = \{0, 1\}, \quad \forall (i,j)$
\[ \nu_{ij} : \text{valuation of buyer } i \text{ for house } j \]

\[ x_{ij} = \begin{cases} 
1 & \text{if } j^{th} \text{ house is allocated to buyer } i \\
0 & \text{otherwise} 
\end{cases} \]

\[ p_j : \text{price of house } j \]

Maximize \[ \sum_{i,j} \nu_{ij} x_{ij} \]

\[ \sum_j x_{ij} \leq 1, \quad \forall i \quad \lambda_i \]

\[ \sum_i x_{ij} \leq 1, \quad \forall j \quad p_j \]

\[ x_{ij} \geq 0, \quad \forall (i, j) \]
Play A Game

• Each of you will put $[0, 10]$ in a pot in a sealed envelope
• $M \leftarrow$ Total money in the pot
• Each will receive $\leftarrow 2*M/#people$

What is the best strategy that maximize your own payoff?
Next Class

- Nash equilibrium concept
- Zero-sum games
- Minimax theorem and LP duality